Physics 208 Exam 1 Name Solution

You are graded on your work, with partial credit. (Be sure to include the proper units in each answer.)
The answer by itself is not enough, and you receive credit only for your work.
Please be clear and well-organized, so that we can easily follow each step of your work.
See the last page of the exam for the formula sheet.

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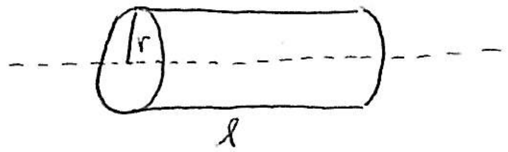
1. We calculated the electric field due to an infinite line of charge, and found that it is perpendicular to the line, with magnitude $E = 2k\frac{\lambda}{r}$. Here $k = \frac{1}{4\pi\varepsilon_0}$ is the Coulomb's law constant. Consider an imaginary cylinder with radius $r = 0.40$ m and length $\ell = 0.25$ m, whose axis runs along the line of charge. The charge per unit length on the line is $\lambda = -4.0$ nC/m, with 1 nC $= 10^{-9}$ C.

(a) (4) Draw a picture below, showing the cylinder and line of charge, with r and ℓ for the cylinder also shown.



(b) (8) Calculate the electric flux through this cylinder due to this infinite line of charge.

One way is to use $\Phi_E = E \cdot \text{area} = 2k\frac{\lambda}{r} \cdot (\ell \cdot 2\pi r)$

with $k = \frac{1}{4\pi\epsilon_0}$, or $\boxed{\Phi_E} = \cancel{2} \cdot \frac{1}{\cancel{4\pi}\epsilon_0} \frac{\lambda}{\cancel{r}} \cdot \ell \cdot \cancel{2\pi r} = \boxed{\frac{\lambda \ell}{\epsilon_0}}$.

The other way is to use Gauss's law:

$\boxed{\Phi_E =} \frac{\text{charge}}{\epsilon_0} = \boxed{\frac{\lambda \ell}{\epsilon_0}}$.

Then $\boxed{\Phi_E =} \dfrac{(-4.0 \times 10^{-9}\ \frac{C}{m})(0.25\ m)}{8.85 \times 10^{-12}\ \frac{C^2}{N \cdot m^2}} = \boxed{-113\ \frac{N}{C} \cdot m^2}$

[other version: $-226\ \frac{N}{C} \cdot m^2$]

(c) (4) Determine the flux through the cylinder if its radius is increased to 1.20 m.

From Gauss's law, same enclosed charge $\Rightarrow$ same flux,

$\boxed{\Phi_E = -113\ \frac{N}{C} \cdot m^2}$ [other version: $-226\ \frac{N}{C} \cdot m^2$]

(d) (4) Determine the flux through the cylinder if its length is increased to $\ell = 0.75$ m.

Now 3 × enclosed charge $\Rightarrow$ 3 × flux:

$\boxed{\Phi_E = -339\ \frac{N}{C} \cdot m^2}$

[other version: $-452\ \frac{N}{C} \cdot m^2$]

2. A charge of $q_1 = +3.0\ \mu\text{C}$ is placed at $x = 0$, and a charge of $q_2 = -6.0\ \mu\text{C}$ is placed at $x = d = 0.6$ m. (Recall that $1\ \mu\text{C} = 10^{-6}$ C.)

(a) (5) Calculate the magnitude F of the force exerted by one charge on the other.

Answer: $F =$ $0.45\ N$

$\boxed{F =} k \dfrac{|q_1 q_2|}{d^2}$

$X \equiv |x|$ below

$= \left(9.0 \times 10^{9}\ \dfrac{N \cdot m^2}{C^2}\right) \dfrac{(3.0 \times 10^{-6}\ C)(6.0 \times 10^{-6}\ C)}{(0.6\ m)^2}$

$= \boxed{0.45\ N}$ [other version: $0.050\ N$]

(b) (15) Calculate the value(s) of x at the point(s) where the force on a small test charge q_0 would be equal to zero. Is there more than one such point? Explain why or why not.

Answer: $x =$ $-1.45\ m$

The force can equal 0 only to the left of q_1, where the force due to the smaller but closer charge q_1 can balance the larger but farther charge q_2.

Then $-\cancel{k} \dfrac{|q_1|\cancel{q_0}}{X^2} + \cancel{k} \dfrac{|q_2|\cancel{q_0}}{(X+d)^2} = 0$ where $X \equiv |x|$ to simplify writing

$\Rightarrow X^2 + 2dX + d^2 = rX^2$ where $r \equiv \dfrac{|q_2|}{|q_1|}$ again to save writing

$\Rightarrow (1-r)X^2 + 2dX + d^2 = 0$.

Use quadratic formula:

$$\boxed{X = \frac{-2d \pm \sqrt{(2d)^2 - 4(1-r)d^2}}{2(1-r)}}$$

with $r = 2$ here

$= d \cdot \dfrac{-2 \pm \sqrt{4 - 4(-1)}}{2(-1)}$

$= d \cdot (1 \mp \sqrt{2})$

$\boxed{= (1+\sqrt{2})\, d}$ since $X > 0$

$= 2.414\ d$

$= 1.45\ m$

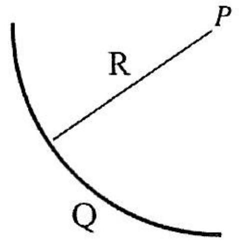
Check:

$(X+d)^2 = [(1+\sqrt{2})d + d]^2$
$= d^2[2+\sqrt{2}]^2$
$= d^2(4 + 2 \cdot 2\sqrt{2} + 2)$
$= (6 + 4\sqrt{2})d^2$

$X^2 = [(1+\sqrt{2})d]^2$
$= (1 + 2\sqrt{2} + 2)d^2$
$= (3 + 2\sqrt{2})d^2 = \frac{1}{2}(X+d)^2$ ✓

Then $\boxed{x =} -X = \boxed{-1.45\ m}$ [-2.90 m in other version]

3. (20) Consider $1/4$ of a full ring of positive charge, as shown in the figure. Its total charge is Q, and the radius of the ring is R. Calculate the electric potential V at the center of the ring, point P, in terms of Q, R, and either the Coulomb's law constant $k = \dfrac{1}{4\pi\varepsilon_0}$ or ε_0.



(To get credit, please set up the integral and **show all your work clearly!**)

$$\boxed{V =} \int k \frac{dq}{r}$$

$$= \frac{k}{R} \int dq \qquad \text{since } r = R \text{ for all } dq$$

$$= \boxed{\frac{k}{R} Q}$$

[same in other version, although geometry is different, except $R \to a$]

4. (20) In class, we started with the electric field outside a conducting sphere with charge q, and then we performed the appropriate (very simple) integral to find that the potential is $k\frac{q}{r}$, just as for a point charge. Now let us do exactly the same for an electric **dipole** consisting of charges $+q$ and $-q$ separated by a distance d, for a point along the axis of the dipole which is a distance r away, with $r \gg d$.

We showed that the electric field due to this dipole has a magnitude

$$E = 2k\frac{p}{r^3} \quad , \quad \text{where } p = qd \text{ is the electric dipole moment, and } k = \frac{1}{4\pi\varepsilon_0}.$$

Starting with this result, and performing the appropriate integral, calculate the potential $V(r)$ at a point (along the axis of the dipole) which is a distance $r \gg d$ away from the dipole. With $\vec{r}$ closer to $+$ charge.
Give your answer for $V(r)$ in terms of k, p, and r.

(As always, please show all the steps in your work clearly. The integral is again very simple!)

$$V(\infty) - V(r) = -\int_r^\infty \vec{E}\cdot d\vec{\ell} \qquad \text{with } d\vec{\ell} = d\vec{r}$$

$$= -\int_r^\infty 2k\frac{p}{r'^3}\,dr'$$

$$= -2kp\int_r^\infty \frac{1}{r'^3}\,dr' \qquad \text{since } \int r^{-3}dr = \frac{r^{-2}}{-2}$$

$$= k\cdot p\left[\frac{1}{r'^2}\right]_r^\infty \qquad \left[\int r^n dr = \frac{r^{n+1}}{n+1}\right]$$

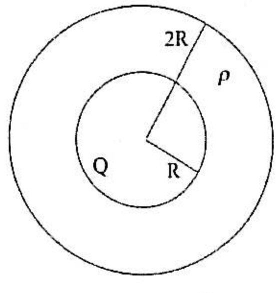
$$= kp\left(\frac{1}{\infty} - \frac{1}{r^2}\right)$$

$$= -\frac{kp}{r^2}$$

$$\Rightarrow \boxed{V(r) = k\frac{p}{r^2}} \qquad \text{if we choose } V(\infty) = 0$$

5. A solid conducting sphere with radius R carries a positive total charge Q. The sphere is surrounded by an insulating shell with inner radius R and outer radius $2R$. The insulating shell has a uniform charge density ρ. (Give all your answers in terms of Q and R.)

(a) (5) Calculate the value of ρ so that the net charge of the entire system is zero.



Answer: $\rho = \frac{-3}{28\pi}\frac{Q}{R^3}$

$\boxed{\text{Volume of shell} =} \frac{4}{3}\pi(2R)^3 - \frac{4}{3}\pi R^3$

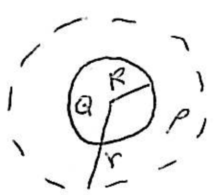
$= (8-1)\cdot\frac{4}{3}\pi R^3 = \boxed{\frac{28}{3}\pi R^3}$

Then $\boxed{\rho =} \frac{-Q}{\frac{28}{3}\pi R^3} = \boxed{\frac{-3}{28\pi}\frac{Q}{R^3}}$.

(b) If ρ has the value found in part (a), determine the magnitude of the electric field in each of the following regions.

(i) (2) $0 < r < R$

Answer: $E =$ 0 [inside conductor $E = 0$]



(ii) (6) $R < r < 2R$

Answer: $E =$ See below

From Gauss's law, with $Q(r)$ = charge inside Gaussian surface,

$$\underbrace{\oint \vec{E}\cdot d\vec{A}}_{= E\cdot 4\pi r^2} = \frac{Q(r)}{\epsilon_0} = \frac{Q}{\epsilon_0} + \rho\cdot\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R^3\right)\cdot\frac{1}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0} - \frac{3}{28\cancel{\pi}}\frac{Q}{R^3}\cdot\frac{4}{3}\cancel{\pi}\left(r^3 - R^3\right)\cdot\frac{1}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0}\left(1 - \frac{1}{7}\frac{r^3}{R^3} + \frac{1}{7}\right) = \frac{Q}{\epsilon_0}\cdot\frac{1}{7}\left(8 - \frac{r^3}{R^3}\right)$$

$$\Rightarrow \boxed{E =} \frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{7}\left(\frac{8}{r^2} - \frac{r}{R^3}\right)$$

$$= \boxed{kQ\cdot\frac{8}{7}\left(\frac{1}{r^2} - \frac{1}{8}\frac{r}{R^3}\right)}, \quad k = \frac{1}{4\pi\epsilon_0}$$

[Check: $r = R \Rightarrow E = kQ\cdot\frac{8}{7}\left(\frac{1}{R^2} - \frac{1}{8}\frac{1}{R^2}\right) = \frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{R^2}$

so $\boxed{E =} \frac{Q/4\pi R^2}{\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$, $\sigma = \frac{Q}{4\pi R^2}$ [surface charge density]

and this is correct just outside conductor.

$r = 2R \Rightarrow E = kQ\cdot\frac{8}{7}\left(\frac{1}{4R^2} - \frac{1}{4R^2}\right) = 0$, as in part (iii) below.]

(iii) (2) $r > 2R$

Answer: $E =$ 0

From Gauss's law, charge enclosed $= Q - Q = 0$

$$\Rightarrow \underbrace{\oint \vec{E} \cdot d\vec{A}}_{= E \cdot 4\pi r^2} = 0 \Rightarrow E = 0$$

(c) (5) Calculate the potential difference between the surface of the conductor, where $r = R$, and the surface of the insulator, where $r = 2R$.

$$V(2R) - V(R) = -\int_R^{2R} E\,dr$$

$$= -\int_R^{2R} kQ \cdot \frac{8}{7}\left(\frac{1}{r^2} - \frac{1}{8}\frac{r}{R^3}\right) dr$$

$$= -\frac{8}{7} kQ \left[-\frac{1}{r} - \frac{1}{8R^3}\frac{r^2}{2}\right]_R^{2R}$$

$$\left(\text{since } \int r^{-2} dr = \frac{r^{-1}}{-1} \ \& \ \int r\,dr = \frac{r^2}{2}\right)$$

$$= +\frac{8}{7} kQ \left[\left(\frac{1}{2R} - \frac{1}{R}\right) - \frac{1}{8R^3}\cdot\left(\frac{4R^2}{2} - \frac{R^2}{2}\right)\right]$$

$$= \frac{8}{7} kQ \left[-\frac{1}{2R} - \frac{3}{16R}\right]$$

$$= -\frac{11}{14} k \frac{Q}{R}$$

or $\boxed{V(R) - V(2R) = \frac{11}{14} k \frac{Q}{r}}$, $k = \frac{1}{4\pi\epsilon_0}$

6. For extra credit.

(a) (5) If a plastic ball with 1 Coulomb of positive charge is placed in your hand, and another plastic ball with 1 Coulomb of positive charge is placed in your friend's hand, and the centers of the balls are separated by 1 meter, what will happen when the balls are released? (Please do not just say that they will repel -- be more specific, and explain!)

Charges will explosively blast apart with enormous force, since

$$F = k\frac{q^2}{r} \sim (10^{10})\frac{(1)^2}{1}\ N = 10^{10}\ N$$

[10 billion newtons ~ millon tons of force].

Hold charge to your side, or you will die!

There are other good and more imaginative answers.

(b) (5) List the 5 types of chemical bonds, and explain which of these bonds involve electric dipoles.

(i) covalent

(ii) ionic

(iii) metallic

(iv) van der Waals (+ -) (+ -) induced dipole

(v) hydrogen [bond] H^+ H^+ O^{-2} molecular dipole [with formal charges shown]